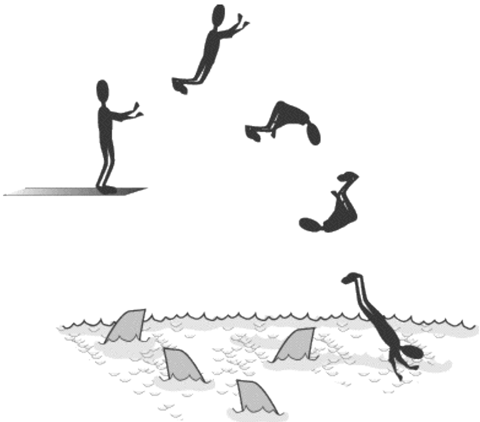
Centre of Mass

When calculating trajectories and collisions, it's convenient to treat extended bodies, such as boxes and balls, as point masses. That way, we don't need to worry about the shape of an object, but can still take into account its mass and trajectory. This is basically what we do with free-body diagrams. We can treat objects, and even systems, as point masses, even if they have very strange shapes or are rotating in complex ways. We can make this simplification because there is always a point in the object or system that has the same trajectory as the object or system as a whole would have if all its mass were concentrated in that point. That point is called the object's or system's **centre of mass**.

Consider the trajectory of a diver jumping into the water. The diver's trajectory can be broken down into the translational movement of his centre of mass, and the rotation of the rest of his body about that centre of mass.



A human being's centre of mass is located somewhere around the pelvic area. We see here that, though the diver's head and feet and arms can rotate and move gracefully in space, the centre of mass in his pelvic area follows the inevitable parabolic trajectory of a body moving under the influence of gravity. If we wanted to represent the diver as a point mass, this is the point we would choose.

Our example suggests that Newton's Second Law can be rewritten in terms of the motion of the centre of mass:

$$\boldsymbol{F}_{\text{net}} = M\boldsymbol{a}_{\text{cm}}$$

Put in this form, the Second Law states that the net force acting on a system, $\boldsymbol{F}_{\text{net}}$ is equal to the product of the total mass of the system, *M*, and the acceleration of the centre of mass, $\boldsymbol{a}_{\text{cm}}$. Note that if the net force acting on a system is zero, then the centre of mass does not accelerate.

Similarly, the equation for linear momentum can be written in terms of the velocity of the centre of mass:

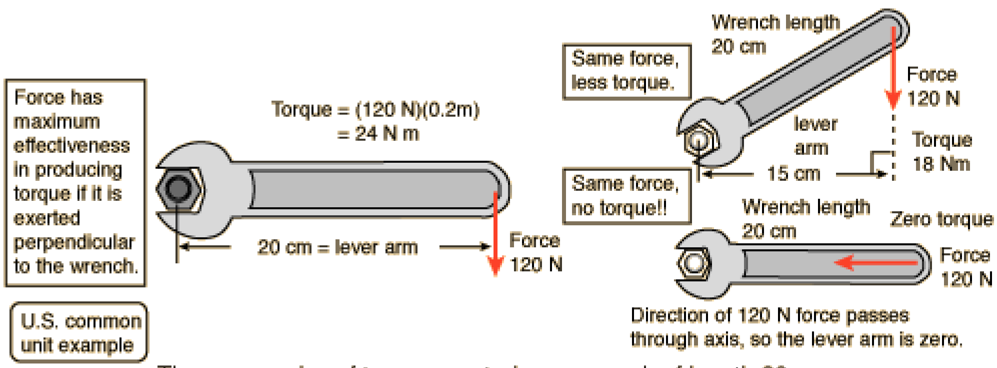
$$\boldsymbol{p} = M\boldsymbol{v}_{\text{cm}}$$

Torque

A torque is an influence which tends to change the rotational motion of an object. One way to quantify a torque is

Torque = Force applied x lever arm

The lever arm is defined as the perpendicular distance from the axis of rotation to the line of action of the force.



Three examples of torque exerted on a wrench of length 20 cm.

Conditions for Equilibrium

An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation. The basic conditions for equilibrium are:

1. Net force = 0	x and y components of force may be separately set = 0.	
$$\sum_i F_i = 0$$	Forces left = forces right Forces up = forces down.	Examples
2. Net torque = 0	The axis may be chosen for advantage to eliminate some unknown forces..	
$$\sum_i \tau_i = 0$$	The sum of the clockwise torques is equal to the sum of the counterclockwise torques.	Examples

The conditions for equilibrium are basic to the design of any load-bearing structure such as a bridge or a building since such structures must be able to maintain equilibrium under load. They are also important for the study of machines, since one must first establish equilibrium and then apply extra force or torque to produce the desired movement of the machine. The conditions of equilibrium are used to analyze the "simple machines" which are the building blocks for more complex machines